DETERMINATION OF ELECTRONIC DISTANCE MEASUREMENT ZERO ERROR USING KALMAN FILTER

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Abstract

Kalman filter has gained popularity in the adjustment of field measurements since it was invented in 1960. Unlike the least squares technique it is relatively not renowned in the adjustment of surveying measurements. This work presents how it can be used in the determination of the electronic distance measurement zero error. First the values of the unknown parameters and their corresponding covariances are predicted. Subsequently, updates on the predicted parameters and covariances are estimated in several iterations. The residuals are estimated to determine the accuracy of the experiment. The estimated electronic distance measurement zero errors for three epochs are 0.015033522833m, 0.0113142487320m and 0.0113121989063m.

Keywords: Electronic distance measurement, Kalman filter, Zero error

Introduction

Surveying measurements are usually compromised by errors in field observations and therefore require mathematical adjustment. In the first half of the 19th century the Least Squares (LS) (Gauss, 1823) adjustment technique was developed. LS is the conventional technique for adjusting surveying measurements. The LS technique minimises the sum of the squares of differences between the observation and estimate. LS has a disadvantage of requiring matrix inversions that tend to slow down the process of adjusting measurements. In this research the use of the Kalman Filter (KF) (Kalman, 1960) technique for determining the zero error of the Electronic Distance Measurement (EDM) will be presented. KF easily resolves the adjustment of field measurements by using a recursive algorithm utilising part of its output as an input for the next iteration (Bezrucka, 2011).

The principles of Kalman filter

The KF is a recursive algorithm (Maybeck, 1979) that estimates the current state of the dynamic system out of incomplete noisy indirect measurements. KF is suitable for both linear and nonlinear processes. "The principle of KF is based on two basic phases of the process: *prediction* and *update"*(Bezrucka, 2011).

KF predicts or estimates the state of a dynamic system from a series of incomplete and /or noisy measurements. Suppose we have a noisy linear system that is defined by the following equations:

$$X_k = A\hat{X}_{k-1} + w_{k-1} \quad (1)$$

$$Z_k = HX_k + v_k \quad (2)$$

Where X_k is estimated state at time k, A is the state transition matrix, X_{k-1} is estimated state for preceding time $k-1$, w is process noise at time $k-1$, Z_k is the measurement, H is the measurement design matrix and v_k is the measurement noise.

Prediction

The previously estimated state $\hat{X}_{k-1}$ can be used to predict the current state at time k, X_{k-1}^{-} as shown by the following equation:

$$X_k^{-} = A\hat{X}_{k-1} \quad (3)$$

$$P_k^{-} = A\hat{P}_{k-1}A^T + Q \quad (4)$$

Where P_k^{-} and $\hat{P}_{k-1}$ are estimated error covariance matrices at times k and $k-1$ respectively; while Q is the process noise covariance matrix.

Update

The Kalman gain K_k is estimated as,

$$K_k = P_k^{-}H^T(HP_k^{-}H^T + R)^{-1} \quad (5)$$

R is the measurement noise covariance matrix. Equation 5 shows that a more precise measurement (i.e. the lower covariance matrix elements) raises its weight (Welch and Bishop, 2001).

$$\lim_{R \to 0} K_k = H^{-1} \quad (6)$$

H is generally a nonsquare matrix, and thus cannot be inverted. Equation 6 should be stated in the following form,

$$\lim_{R\to 0} K_k H = I \tag{7}$$

The a priori covariance matrix approaching zero values means the low weight of the observation and a priori residual (Welch and Bishop, 2001),

$$\lim_{P_k^- \to 0} K_k = 0 \tag{8}$$

Assuming the apriori residual e_k as the difference between the current observation and the expected observation determined in the last parameter estimate is (Bezrucka, 2011),

$$e_k = Z_k - H\hat{X}_k^- \tag{9}$$

Therefore updated estimate of state is determined as,

$$\hat{X}_k^+ = \hat{X}_k^- + K_k e_k \tag{10}$$

and its updated covariance matrix is determined as,

$$P_k^+ = (I - K_k H)P_k^- \tag{11}$$

Application

In order to determine the instrument constant or zero error (k) of a short-range EDM a straight baseline design method was implemented (Ayeni, 2001). Twelve measurements were made using points A, B, C and D (Figure 1 and Table 1).

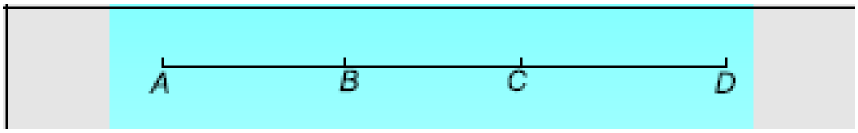

Figure 1. Baseline measurements (Ayeni, 2001)

Table 1. Observations (Ayeni, 2001)

	From	To	Distance(m)
D_1	A	B	101.5107
D_2	A	C	304.2202
D_3	A	D	657.1187
D_4	B	A	10..5204
D_5	B	C	202.7182
D_6	B	D	555.6222
D_7	C	A	304.2297
D_8	C	B	202.7147
D_9	C	D	352.9148
D_{10}	D	A	657.1113
D_{11}	D	B	555.6198
D_{12}	D	C	352.9141

The unknown parameters to be determined are the adjusted distances AB, BC, CD and zero error k. The observation equations are (Ayeni, 2001),

$$D_{1a} = X_1 + k$$
$$D_{2a} = X_1 + X_2 + k$$
$$D_{3a} = X_1 + X_2 + X_3 + k$$
$$D_{4a} = X_1 + k$$
$$D_{5a} = X_2 + k$$
$$D_{6a} = X_2 + X_3 + k$$
$$D_{7a} = X_1 + X_2 + k$$
$$D_{8a} = X_2 + k$$
$$D_{9a} = X_3 + k$$
$$D_{10a} = X_1 + X_2 + X_3 + k$$
$$D_{11a} = X_2 + X_3 + k$$
$$D_{12a} = X_3 + k$$

Where D_{ia} are the adjusted distances; X_1, X_2, X_3 represent adjusted distances AB, BC and CD respectively; while k represents the adjusted EDM instrument zero error. The solution to the stated problem was implemented using the MATLAB programming software.

The solution is, $X = \begin{vmatrix} X_1 \\ X_2 \\ X_3 \\ k \end{vmatrix}$ furnished by equation 10.

The values of $X_{k=0}$, $P_{k=0}$, Q, Z_k, H, A and R are given as,

$$X_{k=0} = \begin{vmatrix} 0 \\ 0 \\ 0 \\ 0 \end{vmatrix},\ P_{k=0} = \begin{vmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{vmatrix},\ Q = \begin{vmatrix} 0.0500 & 0 & 0 & 0 \\ 0 & 0.0500 & 0 & 0 \\ 0 & 0 & 0.0500 & 0 \\ 0 & 0 & 0 & 0.0500 \end{vmatrix},$$

$$Z_k = \begin{vmatrix} 101.5107 \\ 304.2202 \\ 657.1187 \\ 101.5204 \\ 202.7147 \\ 555.6222 \\ 304.2297 \\ 202.7147 \\ 352.9148 \\ 657.1113 \end{vmatrix},\quad H = \begin{vmatrix} 1 & 0 & 0 & 1 \\ 1 & 1 & 0 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{vmatrix} \text{ and}$$

$$R = \begin{vmatrix} 8.4e-7 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 7.2e-5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2.0e-5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7.5e-5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 8.5e-5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3.9e-6 & & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4.3e-6 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3.6e-5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5.0e-5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3.7e-5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3.5e-5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4.2e-5 \end{vmatrix}$$

Z_k is the measurement shown in Table 1. H was derived from the observation equations, while the stated values for $X_{k=0}$, $P_{k=0}$, Q, A and R were assumed a priori.

Results

Three iterations were run to determine the values of X_k^-, P_k^-, X_k^+ and P_k^+ for $k=1,...,n$, where $n=3$ (Table 2). A posteriori residuals were calculated for $k=1$, $k=2$ and $k=3$ (Figure 2). The residuals showed the errors in computing the adjusted values of the unknown parameters. Figure 3 was calculated by summing the absolute values of the residuals for $e_{k=1}$, $e_{k=2}$ and $e_{k=3}$. From Figure 3, $k=2$ yielded the lowest residual value and therefore the most accurate while $k=1$ yielded the highest residual value and therefore the least accurate. From Table 2, 00.015033522833m, 0.0113142487320m and 0.0113121989063m were the estimated EDM zero errors for three epochs $k=1$, $k=2$ and $k=3$ respectively.

Table 2. Estimated results for the unknown parameters and their covariances

$X_{k=1}^-$	$P_{k=1}^-$			
0	1.0498000	0	0	0
0	0	1.0498000	0	0
0	0	0	1.0498000	0
0	0	0	0	1.049800
$X_{k=1}^+$	$P_{k=1}^+$			
101.486066359826	0.100434e-4	0.00868e-4	0.072341 e-4	-0.099439 e-4
202.694571635624	0.00868e-4	0.037717 e-4	-0.019596 e-4	-0.015682 e-4
352.856367289952	0.072341 e-4	-0.019596 e-4	0.101530 e-4	-0.073375e-4
0.015033522833	-0.099439 e-4	-0.015682 e-4	-0.073375 e-4	0.106519e-4
$X_{k=2}^-$	$P_{k=2}^-$			
101.4860663598256	0.0500100	0.000000867778	0.00000723268	-0.00000994191
202.6945716356237	0.000000867768	0.0500038	-0.0000019592	-0.00000156784
352.8563672899521	0.00000723268	-0.0000019592	0.0500102	-0.000007336
0.0150335228326	-0.00000994191	-0.00000156784	-0.000007336	0.0500106
$X_{k=2}^+$	$P_{k=2}^+$			
101.4997768205013	0.100386e-4	0.00867688 e-4	0.0722998e-4	-0.0993895
202.7149962351165	0.00867688e-4	0.0377138 e-4	-0.0195942e-4	-0.0156781e-4
352.8953807316197	0.0722998e-4	-0.0195942 e-4	0.101489e-4	-0.0733329e-4
0.0113142487320	-0.0993895	-0.0156781e-4	-0.0733329e-4	0.1064681e-4
$X_{k=3}^-$	$P_{k=3}^-$			

101.4896268428193	0.0500100	0.000000867514	0.00000722854	-0.00000993696
202.6947247354930	0.000000867514	0.0500038	-0.00000195902	-0.00000156750
352.8600911935466	0.00000722854	-0.00000195902	0.0500101	-0.00000733183
0.0113131173071	-0.00000993696	-0.00000156750	-0.00000733183	0.0500106
$X^{+}_{k=3}$	$P^{+}_{k=3}$			
101.4997788149447	0.100386e-4	0.00867688 e-4	0.0722998e-4	-0.0993895e-4
202.7149962791453	0.00867688 e-4	0.0377138e-4	-0.0195942e-4	-0.0156781e-4
352.8953825410382	0.0722998e-4	-0.0195942e-4	0.101489e-4	-0.0733329e-4
0.0113121989063	-0.0993895 e-4	-0.0156781e-4	-0.0733329e-4	0.106468e-4

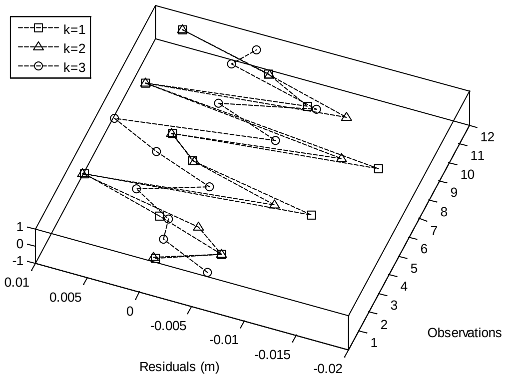

Figure 2. Residuals

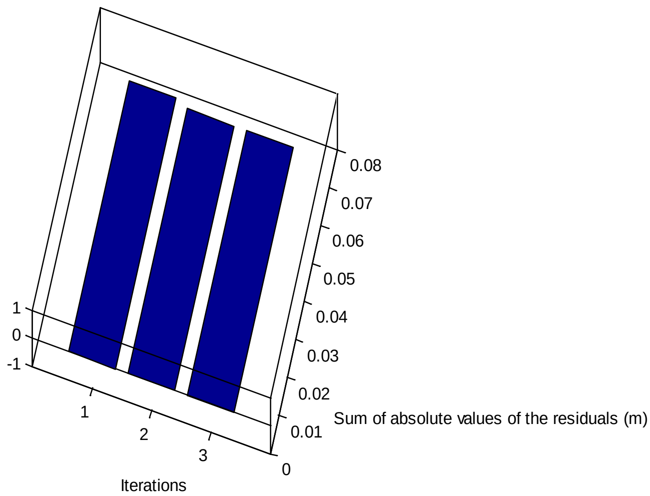

Figure 3. Sum of absolute values of the residuals

Various values of R were used to compute the adjusted values of the actual measurements (Figure 4). From Figure 4, $R(k = 0)$ (whose matrix was given in the previous section) yielded the best fit of the actual measurements. While $R = 10e - 0 * eye(12)$ yielded the least fit of the actual measurements.

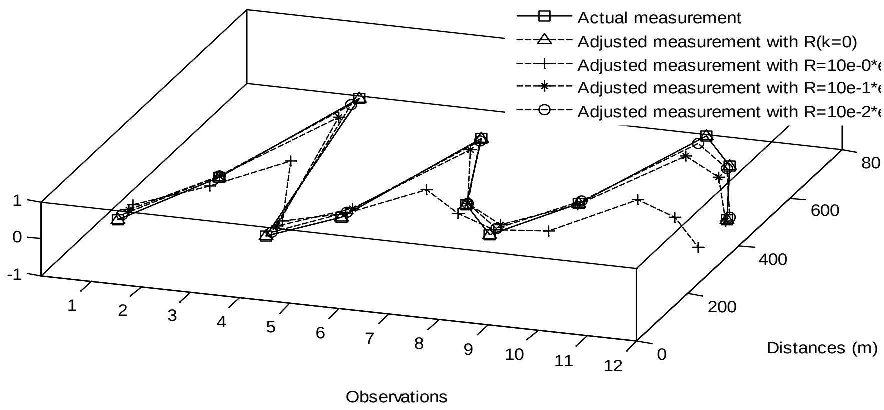

Figure 4. Computed adjusted measurements using various R values

Conclusion

The KF has the benefit of furnishing several solutions in successive iterations as shown in this work. Other advantages of the KF include the possibility of altering the values of the process noise covariance matrix Q and measurement noise covariance matrix R in order to improve the accuracy of the prediction.

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